Astronomy Cast Episode 267 for Monday, May 28, 2012:

Infinities

Fraser: Welcome to Astronomy Cast, our weekly facts-based journey through the Cosmos, where we help you to understand not only what we know, but how we know what we know. My name is Fraser Cain; I'm the publisher of *Universe Today*, and with me is Dr. Pamela Gay, a professor at Southern Illinois University – Edwardsville. Hi, Pamela. How are you doing?

Pamela: I'm doing well. How are you doing, Fraser?

Fraser: I'm doing really well, and I think we want to remind people that they can celebrate the world not ending with us at the end of December.

Pamela: Yes. So we are going on a cruise departing out of Miami in the beginning part of December, second week of December, so hopefully kids will be most of the way out of school for Christmas break, and we are going to spend December 20 at Mayan ruins, stomping around being archeological fans, and celebrating the fact that the world is still around -- and the Mayans built really awesome, big things out of rocks.

Fraser: Yeah. And so if you want to join us you can go to astrosphere.org; there's a link right there on the homepage that links you to information on the "Not the End of the World Cruise," and information on how you can book a ticket on that, and then when you do book a ticket, make sure that you say "Astronomy Cast sent me," and that way we'll be able to sort of gather you together into the special events that we're going to be doing just for Astronomy Cast fans.

Pamela: There'll be live recordings, there's going to be all sorts of fun stuff. Yeah, we really hope that you will be there, that you'll join us, and we can all have a great holiday together.

Fraser: Yeah, it's going to be a nice mixture of relaxation and astronomy, so I think this is going to be fun, so check it out. Alright, let's get on with the show then.

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Fraser: So forever is a funny thing. Today we're going to talk about infinities. That's right, all the different kinds of possible infinities, how you add them, subtract, and use them to think about the scale of the Universe. Alright, Pamela, so you're just ready for some like killer first question, aren't you? Like, what would a four-year-old ask? "Why…?" Yeah, so let's go back to the history of infinity. We're so used to this idea of infinity in…you know, about forever, and about time going on forever, but infinity is actually…it's not really naturally occurring in science, so where did this concept even come from?

Pamela: It's actually kind of awesome -- both "infinity" and "zero" are relatively new concepts in the grand scheme of the Universe, and…

Fraser: Infinity I believe, but zero is a shocker.

Pamela: Zero is the one that's always confusing. So infinity is something that people didn't really think about, talk about until somewhere between 400 and 500 years B.C., and the idea, as far as we know…lots of records have gotten lost throughout the centuries – millennia at this point, but the earliest account that we have comes from Zeno of Elea, and he, basically, was trying to come up with the idea of different paradoxes, different ways of breaking things up, and came across the idea that there were things that…you had finite infinities (and we'll get to this), that you had potential infinities, and these were complicated ideas that left people kind of scratching their heads, and eventually ended up leading to an entire branch of mathematics called "set theory," and I'd like to state here I have not taken set theory, I've taken Calculus, I've taken Relativity, pretty good at tensor math, have not taken set theory, so this is going to be a bit of a reach for me.

Fraser: I've taken a little bit of set theory, but I wouldn't call myself a mathematician able to explain it. But, right. OK, so this is a long time ago, so someone made this leap to say, "OK, let's think about forever, let's think about things that go on from now until forever," and that is something that just doesn't exist in nature.

Pamela: Well, and also thinking about the numbers of things. In terms of: you have a group of things. Can you count them? Can you potentially count them? Are they uncountable? And this eventually led to…I don't know if it led directly because it was a different continent, but in India, you had the way of looking at things where there were "enumerable."

Fraser: Enumerable?

Pamela: Yes, beginning with the letter "e" things, and these are things you can count. So you have a herd of sheep, you can count everything in the herd of sheep, it's a countable number. Then you have numbers that are uncountable: innumerable.

Fraser: " I-N" numberable?

Pamela: Yes, and these are things that are uncountable, but are finite, so the number of hairs on my head, the number of grains of sand on a beach – these are finite numbers, but they're things that you really just can't count, not tractable. But then there's also things that are completely uncountable and they're also not finite, so this is where the idea of "how many pieces can you cut an apple up into?" of "what is the full expanse of the mathematical plane?" And so here you start to get into things that have no boundary to them; they're unbound and uncountable, and that's where the concept of infinity started to come in.

Fraser: Yeah, you can imagine someone saying, "I can count 1, 2, 3, 4… and I could do that forever, but I would never stop counting, I could just keep counting up," and there's got to be some way to describe that.

Pamela: And that also starts to get on to as people started to think more and more about the different types of infinity, the idea that there are countable infinite sets and uncountable infinite sets came up. So you now have… you're starting to talk about different types of infinities. So when I look at the set of whole numbers: {1, 2, 3, 4, 5…}, when I look at the set of even numbers: {2, 4, 6, 8, 10…}, well, both of those are infinite sets of numbers, but the {2, 4, 6, 8, 10…} is smaller than the {1, 2, 3, 4, 5, 6, 7, 8, 9, 10…}.

Fraser: Whoa, whoa -- smaller? But they're both infinity!

Pamela: You have different infinities. So the even numbers is a subset all the whole numbers, but they're both infinite, but you can start counting them, and you're just never going to finish.

Fraser: Oh, OK, so you could take, for example, your even numbers, and you take your whole numbers, and you just start to circle them; you take the 2 on the infinity, and you take the 2 for the even numbers, and then you just match them up, and then you end up with a whole bunch of the, I guess, of the integers of all the numbers that you're not matching up. All the odd numbers, essentially, you're not matching up, and so that essentially gives you a larger infinity. So you can actually have two different, or I guess, an unlimited number of infinities of different sizes.

Pamela: But because we know how the set progresses: 1, 2, 3, 4, 5, 6, 7... 2, 4, 6, 8, 10…these are called countable infinities. Now, at the same time, you can imagine that you take all the numbers between 1 and 2, so 1.1, 1.2, 1.20579812, and you just keep adding decimal points, now you have the real numbers: 4/3, "pie" is a real number, "E" is a real number…and these numbers, with all these decimal points added to them, you can create an infinite number of real numbers between 1 and 2, and then there's another infinite between 2 and 3, and then if you try and come up with a set of all the real numbers, that's an uncountable set. There's no way that I can sit here like I do with whole numbers that go 1, 2, 3, 4, 5. I can't do that with real numbers.

Fraser: Right.

Pamela: And so now you have an uncountable infinite set, so when you start looking at infinities, there's multiple types of infinities, and they're not all the same, and so this has led to entire branches of mathematics.

Fraser: So where did the little infinity symbol come from? (I'm making it in the camera)

Pamela: [laughing] Well, it's one of those things…to me, it's always looked like the number 8 fell over sideways and had a bad day. A lot of infinity signs are sort of an elongated number 8, and there's actually a little bit of arguing over exactly what the origins of it…John Wallis, who was a theologian, is credited in the mathematical literature with coming up with the symbols. This was in 1655, in *De sectionibus conicis* (sorry, I can't pronounce Latin well), and it's argued over whether or not the symbol came from the numbers for "1000" in the Etruscan numeral set, where it was basically the letter C, a straight line, a backwards letter C…if you smush all of that together, it sort of looks like a sideways number 8, sort of like the infinity symbol. There's others that say that it's a mutation of "omega," which is the end of the Greek alphabet. So you have "alpha" as the beginning, "omega" as the end, if you take the lower case omega, it's like a curly W. Well, curl that little W in on itself -- maybe you get the infinity symbol. So what he used for inspiration is still up for debate, but it was definitely John Wallis that came up with the symbol that we now use.

Fraser: And so, I mean, as we said, you can count to infinity, or I guess you can never count to infinity, you can *attempt* to count to infinity, but infinity actually has a role to play. You know, mathematicians, physicists use infinity all the time in math to help with certain ideas.

Pamela: So it really, actually comes out left and right, and along with infinity, you have to have the infinitesimal, so this is the idea that it's something so small that it's almost zero, but not really. And so you can have different-sized infinitesimals, just like you can have different-sized infinities, and it starts to get a little bit confusing to deal with at times, but you have to use both partner ideas. And this actually builds on what we were talking about last week when we talked about Archimedes, and Archimedes, when he was trying to figure out how to calculate the area under a curve, started with the idea that you take the curve and you divide it up into a bunch of sections that you can measure the area of -- a bunch of small rectangles is what gets used when we teach it in modern mathematics classes, and by making the rectangles smaller and smaller you can better approximate the area under the curve by summing all of these together. Well, if you bring in the idea of the infinitesimal, such that each rectangle is basically coming down to zero width (or at least as close to zero width as you can get), and then you're adding up all of these together by summing all of these infinitesimally-small rectangles together, you have a continuum of rectangles that give you an accurate measurement of the area under the curve. This was the idea that got us to integral calculus and differential calculus.

Fraser: And those are used…

Pamela: Everywhere.

Fraser: …all the time by scientists.

Pamela: Yeah, and so this dates back to…well, Newton and Leibniz had to come up with calculus in order for us to have pretty much all of modern physics, so Leibniz was working on infinitesimals; Newton was working on physics. The two of them together both came up with differential and integral calculus, using somewhat different and overlapping applications. And with this idea that you can sum over various things -- it allows you to start thinking in complicated ways, so you start summing functions from zero to infinity, from negative infinity to positive infinity to get the: "What is the area under the curve? What is the rotation? What is the summation over a different set?" There's lots of different applications where adding up all the little pieces starts to mean something.

Fraser: Especially when you're able to slice up what you're looking at, as you say, infinitesimal pieces, so you're taking a set amount of information, but you're slicing it up so many times that you get an approximation, which is a very accurate mathematical estimate of this thing. I know physicists use this process to estimate, in some cases, the force of certain…like, forces in physics and things like that.

Pamela: Well, and the place that calculus most abruptly hits you over the head the first time is just looking at something as simple as velocity and acceleration, where you look at the changes in the shapes of the different curves, of how far have you been displaced, what is your velocity over time, and you can use the derivatives of these different values to get to the next value, or to figure out how far you've gone if you did the integral to go backwards, and so the relationships between distance traveled, velocity, and acceleration all require the use of calculus to figure out changes over time, and to figure out complete distances, complete velocities, and things like that.

Fraser: So I think there's a great analogy that's been used to sort of understand these different sizes of infinity. We talked a bit about that about how you could be matching up the even and the odd numbers, and so there was the paradox, the hotel paradox. Are you aware of this?

Pamela: So this is Hilbert's Paradox. This is a paradox that was discussed in the early part of the last century, where you can imagine that you have your normal hotel that has a finite number of guests. That's easy to deal with. Now, instead imagine that you have an infinity of new guests coming in, so you basically now have a countable infinity, where one comes in, you put them in a room, then you just keep adding the rooms as you add guests. This is the same idea as there's a variety of different artwork trying to explain an infinite universe where you just keep taking an extra step, or shooting an arrow, and where that arrow lands…well, you've now just made the Universe bigger. Well, in this case, you just add a guest and the hotel gets bigger, so this is still a countable infinity in the sense that it's a whole number of rooms, the same way we have integer numbers that are a countable set.

Fraser: And the hotel is full.

Pamela: And the hotel is always full because you're adding rooms for every guest, and so it's basically the set of whole numbers. Now, it starts to get a little bit yuckier when, instead, what you have coming up to the door is a whole series of coaches, and each coach now includes an infinite set of guests. So now I have a countable set of coaches, maybe (you can always say that there's an infinite number of driveways that each contain an infinite set of coaches), but now, because I have all of these coaches that each contain infinite guests, that's the same as the idea of there's an infinite set of numbers between 1 and 2, and an infinite set of numbers between 2 and 3 that are the real numbers, so this was a way of taking what, to me, seems perfectly natural because I've been thinking about this part of set theory for a long time, but trying to make it sensible in a paradigm that many people have dealt with, which is the hotel syndrome. So you can imagine the mad construction builders on the roof of the building just adding rooms every time there's a new guest. It's a countable infinity. You can imagine the coach that has an infinite number of guests within it, and these coaches keep pulling up, and now that becomes uncountable.

Fraser: Woo.

Pamela: Yeah, it hurts.

Fraser: Yeah, but there's actually a great… I think it's on "Horizon." If you do like a Google search on "infinity" and "Horizon…"

Pamela: BBC "Horizon."

Fraser: …BBC “Horizon,” yeah. There’s a great whole episode just on infinity, and they cover the hotel paradox in it quite nicely, and it’s pretty great, so I highly recommend that. So I think where we just sort of need to take this conversation now is to apply this into astronomy and cosmology. We’ve understood sort of “what is this concept of infinity?” How does this play out in astronomers thinking in sort of the scale and size of the Universe?

Pamela: The first place it really cropped up was with another paradox, in this case, Olbers’ paradox. And Olbers was a philosopher who went outside and basically looked up -- and this is going to sound strange, but he realized the sky is dark, and this is problematic. And again, this was happening in the last century. He actually worked on the paradox in the 1800s. And looking up and realizing that the sky is dark had a couple of different consequences because, up until then, we as a society in our philosophy of thinking, had come to the conclusion that the Universe is infinite in size, and if it’s infinite in size, that means that every direction I look, no matter where I look in the sky, my path of my vision is going to end in a star. Now if you start from the premise that the Universe is also infinite in age, that means that the light from every single one of those stars has had time to reach us here on the planet Earth. And when we look into the sky, we should see -- no matter where we look -- the light of a star, and so the entire star should glow with the light of a million billion billion suns, and we don’t see that. And so this implied that our Universe has to be either finite in size, finite in age, or finite in both, and this was a new way of looking at things where it was math, not religion, that was placing limits on the Universe.

Fraser: And, but I know that, sort of, the way that ended up getting resolved was, I mean, partly definitely being limited in age.

Pamela: Yes, we do know the Universe is limited in age. We’re still working on going beyond that is the crazy part.

Fraser: Right, well, and I guess that’s the next step of this conversation, so but I think that’s a fantastic understanding of the Universe for him to make, that he looked up, looked around and said, “Why do I not see stars everywhere in the sky? Why is the whole sky not as bright as the surface of one gigantic star because wherever you look there should be photons streaming from stars? Even if they're infinitely far away, you're still going to get some stars." It's really hard to wrap your mind around.

Pamela: It's so simplistic though.

Fraser: Yeah. No. Absolutely. That is the implication of infinity, that if there's infinity, then there would be something everywhere. Right?

Pamela: And it has to be infinity in both universe and time.

Fraser: And any time you break either one of those…and you don't need to have the whole sky be a star.

Pamela: And you can break both of them at the same time. So all he did, basically, was get rid of an infinity or all of them, so that left a whole bunch of combinations for scientists to spend the next decades and decades working on figuring out.

Fraser: Right, and so as you said, now thanks to Hubble, thanks the Cosmic Microwave Background Radiation, we now are able to place a finite idea of the age itself of the Universe at 13.7 billion years, but the answer of whether or not the Universe is finite or infinite is still an unknown.

Pamela: Yeah, this is one those things that really bothers us. So it was originally hoped that when we launched the Wilkinson Microwave Anisotropy Probe, when we launched WMAP, that we would be able to accurately measure the geometry of the Universe, and it was also thought at the time that there was no acceleration value to how the Universe is expanding, so we had this great vision in the mid '90s that we were almost there. We'd be able to figure out if the Universe was infinite or not based strictly on a geometric argument. So if the Universe contains a high enough density of material, then the light from that material will have a chance to travel all the way around the surface of the Universe, where everything is gravitationally bound. Eventually, the entire Universe will have a giant crunch, so this was one of the fates of the Universe. Then you also had the idea that you could also have a geometrically flat universe, which is the idea of an infinite Euclidean plane, except you're now thinking in four dimensions because you have time, and the three normal spatial dimensions. And then there's also an open idea where if there wasn't enough mass, the Universe would continue to accelerate -- not accelerate, the Universe would continue to expand apart forever. Now, what we've discovered is we have a slightly more confusing situation: the Universe has dark energy. This is an acceleration, something, energy, force, pressure…and this dark energy is causing the Universe to expand apart -- still doesn't rule out the idea of an infinite universe, just makes it a little bit harder to calculate. And looking at the data from the Wilkinson Microwave Anisotropy Probe (WMAP), we're finding that the geometry is completely flat to within .5% of what we can measure. This adds up to: could be infinite…probably infinite…but could also just be really, really, really big.

Fraser: How big?

Pamela: Well…

Fraser: Like, I guess what you're saying, then, is because it's seen as flat, we're not seeing the curvature of it that would tell us that it is finite to a measurable level. We're saying it is definitely flat to the point that we can't tell if it's infinite or finite.

Pamela: And the limits we're putting on it are such that if the Universe is infinite in size, the part of the Universe that we live in is only a few per cent of the total Universe, and when I say the part we live in, I mean the part that we can see, the part of the Universe from which light has had time to travel since the Big Bang to the planet Earth, and we're able to observe it, and so when there's still the possibility that we're only seeing a fraction of the Universe, all we're doing is putting limits on the size of the complete Universe, and it's unclear if we'll ever actually be able to distinguish between an infinite and a finite universe, but there are potentials that if the universe is finite, then we'll be able to measure it.

Fraser: So would a better measurement of these differences in the Cosmic Microwave Background Radiation give us a more precise understanding of that minimum size of a finite universe?

Pamela: It's not so much the better understanding of the geometry as with missions like the Planck mission, which is currently measuring the Cosmic Microwave Background, there's the potential that we'll be able to see places where light from one place in the Universe is being seen in two different places in the sky, and if we can find these places where essentially the Universe has had the chance to wrap around itself, where the light has had a chance to wrap around itself and come at us from two different directions, perhaps that will -- not perhaps, that *will* put limitations on how big the Universe can be.

Fraser: Whoa! So…this is crazy! So if you…in other words, if you are able to look at the Cosmic Microwave Background Radiation of the Universe precisely enough, you should be able to see features mirrored in this microwave. You should look in one direction of the sky and see certain features, and then turn around and look at other features and see those same features on the other side of the sky telling you that the Universe is finite in size, and that if you go far enough in one direction, you will come back around from the other direction.

Pamela: And one of the topographies that's preferred right now is the four-dimensional hyper torus. So this is the idea that, essentially, we live on an expanding doughnut, and when you have a doughnut, the light on the doughnut is able to follow Euclidean geometry, such that two rays of light always stay parallel to one another, and it can either wrap through the hole of the doughnut and the outside of the doughnut, and come back to where it started, or it can race around the surface of the doughnut where you put the frosting down, and two rays will always stay parallel to each other no matter how you put them down on the surface of the doughnut, and you can mathematically expand this to more different dimensions, and maintain the idea of parallel rays, get the whole sucker expanding, and you can see how light from one of the sprinkles on the surface of the doughnut can come out one point from two different places on the sky.

Fraser: So what are the implications, then, if we deal with an infinite universe? What does this mean for us as human beings here on Earth?

Pamela: Well, I think we still live in such a small fraction of that universe that it really doesn't have any implications, but it does mean we can figure out the answers to questions, so you can say to the small child, "Yes, the Universe is finite or infinite. " It starts to bound our ideas for the Big Bang theory. It just puts limitations on ideas.

Fraser: No, but if it's infinite, if it's infinite…

Pamela: Well, if it's infinite, then it's still another different way to now solve an *un*bounded mathematical equation. It's another way to look at the theories.

Fraser: But, I mean, there's mind-bending repercussions.

Pamela: It's a philosophical implication more than anything else because now you're broken with the concept of the entire Universe started out as an infinitely small point that exploded into existence, expanding all points at the same time, no center to this, and that that was an infinite universe expanding from an infinitely small point, not expanding from, that's…we don't even have the language to say this!

Fraser: I know! I know! So how do you…and infinite expansion in…

Pamela: You go from everything infinitely compacted together to everything…

Fraser: …infinitely large.

Pamela: Yes, and…but infinite in all points, and suddenly math hurts far more.

Fraser: Yeah.

Pamela: It's the same problem people have with Relativity.

Fraser: It's worse that that because then you have infinite numbers, we had this conversation about multiple dimensions that if you have an infinite universe, then anything that can exist will exist, and it will exist an infinite number of times, and so there will be an infinite number of sun-like stars out there, and around that infinite number of sun-like stars there will be an infinite number of planets that kind of look like Earth, and in fact, there will be an infinite number of planets that look exactly like Earth, that happen to have evolved a million creatures, some of which look like me and Pamela, and they will be recording an infinite number of Astronomy Cast episodes right now.

Pamela: Now you add to this the problem of the potential for "multiverse," which is the idea that our universe may be one of many different universes that exist in, perhaps, a quantum foam, but…perhaps branch off of one another. No matter how they come into existence, there's the potential for each of them to have different values for the different physical constants, so different value of gravity, different mass of the electron, different finite structure constant, so now you're looking at an infinite number of infinitely-sized universes. Now, this gets back to the whole "real number problem" where there's an infinite number of numbers between 1 and 2, and an infinite number of numbers between 2 and 3, and so now what you have is infinite size within our universe, and there's an infinite number of universes and it's turtles all the way down, but it's not. It's just math, and it hurts.

Fraser: Right. Yeah, there's a few very simple mathematical formulae that would sort of wrap this all up into a nice, tidy bow, but the philosophical implications are mind-bending. So then what about time? Is time infinite?

Pamela: The way, unfortunately, that we have to deal with time within our universe is there was a T=0, and we can't go before T=0, so within the bounding box of the Universe that we exist, time has a moment zero, unless you go with some of the more complicated cosmologies that don't actually have a moment of "big bang," that are more of expanding and contracting ideas, but for the majority of the cosmological ideas, you do have a T=0.

Fraser: But then, what about the other way?

Pamela: Don't know!

Fraser: Will you have a T=infinity?

Pamela: Well, currently, where our understanding is that we live in an accelerating-apart universe, flat geometry or not, if the sucker's expanding apart, it's never going to come back together, so there's not going to be an end to the time, but then there's also ideas of perhaps there's going to be the chance that our universe and another universe touch and combine, and so that would essentially bring time and our universe to an end, and start something new. So this is…think of two universes as soap bubbles that merge into one, or our universe could pop, and these are all terrible ideas, but they're not disallowed by physics.

Fraser: Right, but I guess, what I'm saying is when you think about…let's say that isn't the case, and we're just going to deal with the situation that we have, which is this accelerating expansion of the Universe…

Pamela: It's unbounded.

Fraser: It is unbounded into the future, and then when you really think about that, think about the absolute fortunate time that we happen to be here in this tiny fraction when complicated life and energy and all these things are possible, because for the vast majority of the lifetime of the Universe it's just going to be a rapidly-accelerating super-particles and energy.

Pamela: And eventually, if protons do decay, we'll eventually be a soup of nothing but energy.

Fraser: Nothing but energy expanding…that you can never have any energy differential. There will never be anything else, there will never be life, there will never be, you know, anything -- internet…

Pamela: The future looks bleak.

Fraser: …television, yeah, and the fact that we live in this moment at the very beginning is quite cool, but then that's the, what is it? The greater anthropic…lesser anthropic, lesser anthropic principle, right, which is that we wouldn't be here to observe it if it wasn't possible, so... Well, that was really cool, and I hope everybody's minds have been sufficiently blown this week by the concept of infinity. It is super-cool to think about his stuff, and I highly recommend, like I mentioned, search for this BBC "Horizon" documentary on infinity. It's great and covers a lot of these concepts.

Pamela: And set theory is the thing to look at if you are into math and you want to understand more of this sort of thing.

Fraser: Fantastic! Well, thank you very much, Pamela, and we will talk to you next week.

Pamela: Sounds great, Fraser. Talk to you later.